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A class of relativistically rigid proper clocks

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Abstract. The principles of proper time-keeping are considered and it is noted that an ideal solution may be represented by a system of helical null lines in 4-space. This construction is translated into 3-space and it is shown that it may be interpreted as a self-contained and coherent system of waves having a well defined boundary. These waves befit the de Broglie components which interfere to form a rotating particle. Three configurations are considered. In each of these the localised wave system has two components of energy, one of which is at rest in the chosen frame and the other is circulating relative to it.

These proper clocks appear to have very many of the properties of fundamental particles of matter and yet they are formed from non-particulate waves. It is finally suggested that the de Broglie waves in these particular configurations may be intrinsically of the same substance as electromagnetic waves trapped in an unfamiliar rotating configuration following a cataclasmic event which has squeezed the fields together into a condensed, rotating state which is non-linear in its radial geometry and from which they cannot escape. The stable entities are an example of pure non-particulate matter which may also be applicable on a cosmic scale.

1. Introduction

In order that the laws of physics may hold on any scale it is a fundamental requirement that matter, even on the smallest scale, must observe the keeping of proper time. It appears, therefore, that proper time-keeping must be built into the nature of matter itself and as the speed of light is invariant in the local frame, the units of proper length may also be associated with the same phenomenon. A natural proper clock must be such that it is endowed with relativistic rigidity in order that it may survive acceleration and still keep its own proper time in a different velocity frame. It must never run down and, considered as a system, it must have an infinite Q. Clearly, the cumbersome mechanisms of man-made clocks can only be approximations to the ideal, but their close rivals, the pulsars, despite their scale, are almost equally good approximations. We have set out to see if there could be a mechanism, operative on any scale, which may act rigorously as a proper clock and we give, in this paper, a possible mechanism which may, at least, be similar to that employed by nature. The clocks which we postulate are not composed of discrete particles but are themselves individually particulate. They are a form of pure matter having rest mass, angular momentum and inertia and even if a particular model in the class is entirely hypothetical it still may have applications as a powerful tool in analysis.

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2. Light signals as a basis of proper time-keeping

It has been shown by Fokker (1965), and others, that local light signals reflected between imaginary mirror surfaces having parallel and invariant spatial separation may be used to define the properties of a proper clock. With this we would entirely agree but the model, shown in figure 1, is severely lacking in two respects. One of these is the omission of a mechanism to ensure that the reflecting surfaces remain at invariant (relativistically rigid) distances and the other, which is intrinsically linked, is the crudity of the concept of the light signal.

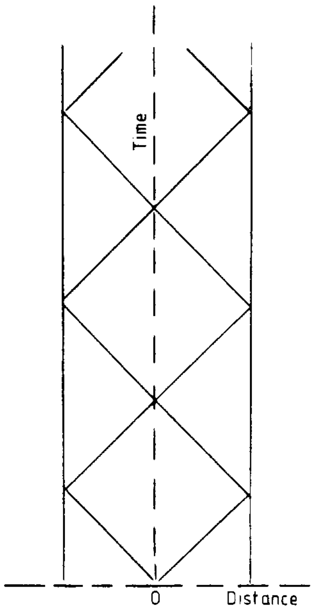

Figure 1. The Fokker clock.

If figure 1 is projected into 3-space it will be seen that it requires the existence of physical reflectors which must somehow know that they are to readjust in position to preserve the precise separation $c\tau$ where τ is the unit of proper time. This cannot be achieved with the mechanics of ordinary material for it requires the velocity of sound to be the velocity of light. If the reflectors are not made of tangible matter they must also be accountable within the framework of the matter of the clock but one may conjecture that, in these circumstances, the same framework could give rise to total internal reflection progressively throughout its whole framework and not just at the boundaries. This would maintain the principles in figure 1 but modify the world lines of the light signals. We also note that in 3-space there has to be a source of the signals which rebound within the system and Fokker's clock gives the impression that these signals are infinitesimally short flashes. If the signals can be suitably contained and figure 1 is given cylindrical symmetry about the central axis then the light cone emitted from O may remain trapped indefinitely and regularly repeat its geometry after reflection provided there is no loss or dispersion in the system, but nature must then provide infinitesimal pulses from an unknown source and decide where to reflect these pulses in order to achieve the basis of its most fundamental properties. The physics of the situation in these terms is unaccountable, even on quantum postulates, and, in particular, it is clear that the 'light signal' has been insufficiently defined.

Figure 2 is similar to figure 1 but now shows (broken) another representative set of lines. The world lines of the light signals now describe the history of the events corresponding to the emission of representative phase components of a continuous monochromatic wave. For simplicity, only two such events are depicted, corresponding to the history of a portion of a wavefront of phase zero (full line) and the history of a portion of a wavefront of phase π (broken line). These successive 'crests' and 'troughs' are sufficient to illustrate the continuous wave nature of the signal which is trapped in the region of space demarcated here in spacetime.

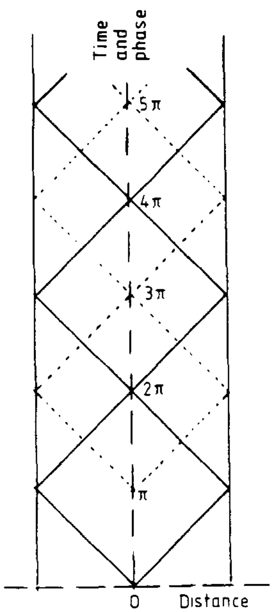

Figure 2. A clock containing a trapped continuous wave. The full lines denote the history of crests and the broken lines represent the troughs.

It will be seen that, provided the reflecting surfaces are maintained at relativistically invariant distances, the events on the proper time axis are always phase congruent, or cyclically conjugate across the axis, subject to the boundary condition at 0. Conversely, if the signals from all directions which arrive at the time axis at any instant carry identical or precisely conjugate phase information and this situation holds for all values of phase throughout all cycles of the wave, then the excursion of the null geodesics in 4-space must be invariant. Therefore the projections into 3-space must also be invariant in proper length and wavelength, and, furthermore, the cycling of the phase will be a correct indicator of proper time.

This same result may be obtained from the principles of interferometry. It is important for we may now rephrase the requirements for a proper clock. The system must be lossless and the phases of the waves must always remain congruent or conjugate on the time axis. That is to say, the original wave must be locked in phase to itself upon the axis of proper time ($x=y=z=0$). If this can be achieved, the system will automatically fulfil the requirements of proper length and relativistic rigidity.

Bearing in mind these simple but practical requirements, we will now endeavour to see if there is a construction which may achieve their satisfaction with the minimum of building material. Noting that fundamental particles behave as proper clocks, not only those with definite half-lives but also the proton and electron, we now imply a wave nature to the interior structure of the particles and phase relationships within a trapped quantum.

This is an important point which we will reiterate in two complementary ways: (i) proper clocks cannot be entirely imaginary but must be composed of elementary matter, and (ii) the elementary particles of matter must themselves be proper clocks in order that they may obey the laws of physics.

Thus we are not restricted to light signals for the operation of proper clocks and we may equally consider de Broglie waves locked in phase in a similar manner. De Broglie mechanics, unlike Schrödinger's, associates an intrinsic wave periodicity with the rest energy of the system and we are encouraged by the important work of Mackinnon (1981a, b) in which he developed a 'stationary' solution of de Broglie waves which, unlike the more usual probability (Schrödinger) analysis, is uniquely located in space. It has the form

$$\psi = [(\sin kr)/kr] \exp(\mathrm{i}\omega t - k_0 x) \tag{1}$$

and befits a non-dispersive wavepacket for a free particle of mass m travelling in the $+x$ direction at a velocity v, where $k_0 = mv/\hbar$ and $\omega = m^2c/\hbar$. This solution, which is consistent with the classical description of a particle, is localised around a phase-locked centre but has side lobes in the form of concentric shells stretching to infinity and takes no account of rotation. Jennison (1983) pointed out that, if it were to account for primary matter, it would have to exhibit angular momentum and he developed Mackinnon's equation to give a confined equatorial system of de Broglie waves in

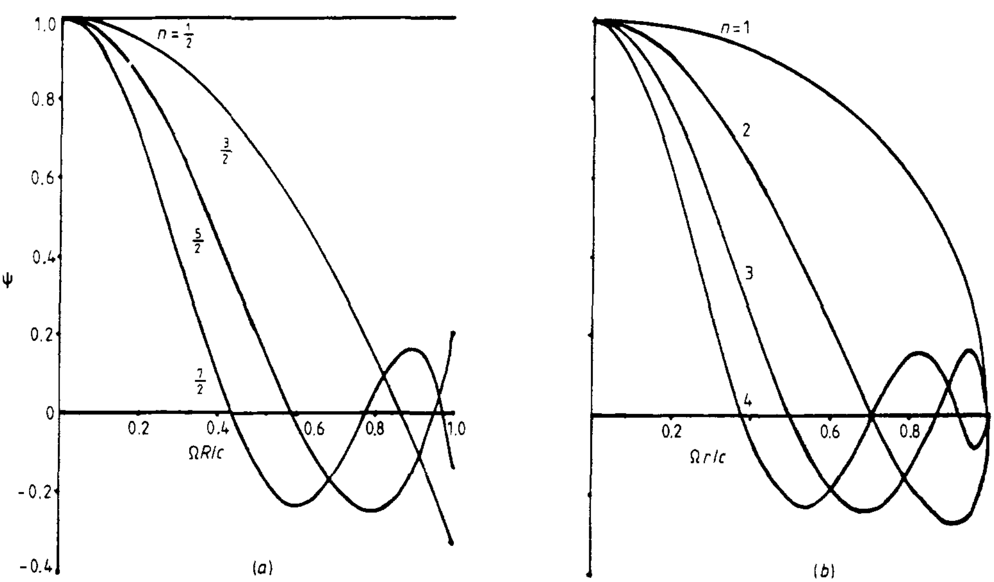

Figure 3. Solutions of equation (2). Plotted for (*a*) odd and (*b*) even half-integral values of *n*.

rotating matter:

$$\psi_{\rm rot} = \frac{\sin[2n \sin^{-1}(\Omega r/c)]}{2n\Omega r/c} \exp(i\omega_{\rm wave} t) \tag{2}$$

where Ω is an angular velocity, r is a radius in the equatorial plane and n is an integer. This equation describes an equatorial system formed by the phase-locked interference of de Broglie travelling wavelets and has solutions which are plotted in figure 3. It refers to a non-linear system in which the matter formed by the interference of waves on curvilinear trajectories appears as a phase-locked rotating structure. The resulting matter is restricted to within the radius at which it is travelling at the speed of light relative to the rest frame of the centre. This does not preclude the possible presence of a field system (e.g. a static field) beyond the limiting radius of the matter, provided such a field is ultimately attributable to the rest energy contained within the radial limit describing the matter.

We will now make one temporary postulate which will, in our opinion, be justified later. We postulate that a real significance may be attached to the helical solutions of Minkowski's differential equation which are described below.

3. A construction in spacetime

Consider a construction in spacetime which has the form of a helix having an axis parallel to the proper time axis at such a spatial distance that the helix itself regularly intersects the proper time axis. Let the pitch angle be $1/c$, identical to that of a light cone in Minkowski diagram. The helix is then a helical null line corresponding to a point on the wavefront of a yet unknown disturbance which moves at the speed of light relative to $xy(z)$ space. It is a solution of the Minkowski (1908) equation

$$dx^2 + dy^2 + dz^2 = c^2\, dt^2. \tag{3}$$

An isolated helical null line of this type may not exist in nature but we shall see that we can ascribe to it the property of a component wave of an integrated structure, somewhat in the manner of the cisoidal Fourier components of a Fourier integral (Jennison 1961). We now introduce the concept of a phase-locked set of similar helices and again one may use the analogy of the Fourier components which are locked in phase at the origin of the function. If one takes a coherent set of helices distributed in azimuth around the time axis so that they are all in contact with the axis at $t = 0$ and the helices are all of the same phase, i.e. are common to a single wavefront, then these helices will all return to the $t = 0$ axis at regular intervals and remain phase-locked for all time. If a physical solution of this system can be found, the helices appear to form proper time calibrators of the otherwise unscaled time axis, i.e. they will behave as a proper clock. Furthermore, the limit of the outward excursion of an identical set of helices is also an invariant and the projection into three-dimensional space is therefore a calibration of proper distance measure in that space.

Can this phase-locked system of helices have a recognisable form in 3-space and if so, is the configuration stable? We will consider first the projection of the helices into xy space orthogonal to the time axis. Each helix now appears as a circle touching the central axis $x = y = 0$, as shown in figure 4 which depicts four out of an infinite set of such circles. Each circle represents the locus of a point on a wavefront and each

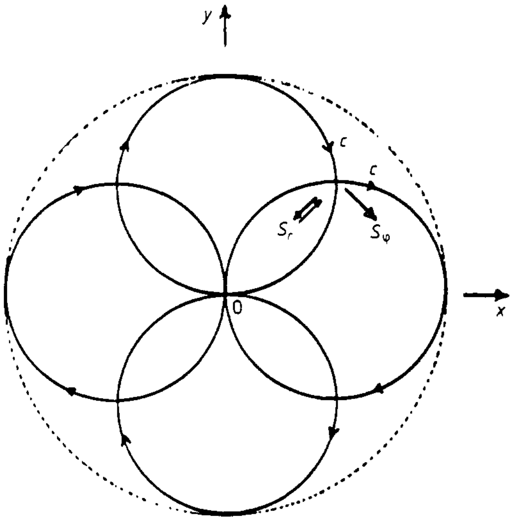

Figure 4. The projection into xy space of four phase-locked helical null lines. The four inner circles represent the loci of coherent phase-locked wavefronts travelling at the speed of light. The velocity vectors combine symmetrically to give a group velocity which circulates in the outer parts and a non-rotating potential distribution in the inner regions. The xy space is later interpreted as a rotating rest frame.

point on a given circle has a velocity of $\c{c}$ where the speed of the disturbance is the usual speed of light but the vector constantly changes direction in xy space to produce a path of constant curvature. If the helices are locked to the *same* phase at the origin, the phase of the wave on the circular path at any instant is the same for all points at the outer edge of the complete system. Similarly the phase at any circle of intermediate radius centred on O has, at the same instant, the same value on all outgoing rays and another related phase, which is identical for all re-entrant rays, but see § 8.

We now recall that all the rays are simply components which, separately, may not be real observables but which may combine coherently to give a real observable, in the manner that the component TEM waves travelling at c in a waveguide combine to give the resultant TE wavegroup which is a real observable of velocity $c_g \leqslant c$.

Vector addition of all the component rays in the case under consideration shows that the group wave is beautifully ordered with the combined group velocity entirely tangential and linearly proportional to the distance from the axis in the range $0 < r < R_0$ where $R_0 = \c{c}/\Omega_0$ and Ω_0 is the angular frequency of the ordered rotation of the group wave system about the centre O. The fastest motion occurs towards the limb where $\Omega_0 r \rightarrow \c{c}$ but towards the centre another phenomenon occurs where the radial components of the rays interfere without rotation, i.e. the radial components combine to form a system which is at rest in the chosen spatial frame. As there are no losses in the system it appears to have an infinite Q, which again indicates an essential feature of an ideal proper clock. However, we must establish whether or not these features can also hold in three spatial dimensions before assuming that such a clock can exist as a physical entity.

Figure 4 shows projected helical null lines in the equatorial plane. It will be observed that equal wave distances along these curved trajectories result in interference at their intersections which is non-linear in terms of the distance measured radially; we will return to this later. Figure 4 would be exactly the same if we were to consider the continuous reception and emission of a disturbance to and from the origin throughout an arbitrary number of clock periods. By analogy with the propagation of light one might then refer to the projection as a ray path. By further analogy with the emission of an electromagnetic signal in an inertial system, we consider that the ray must have a wavefront extending in the third dimension whereby an energy density and a flux may exist in three-dimensional space. Although a single helical null line, even in three-dimensional form, is unlikely to be self-supporting, one can model this form and then integrate it about the polar axis to examine the properties of a symmetrically balanced system.

Before doing so, however, it should be noted that we will be discussing the most elementary systems with a single central node which acts as a source and sink for the travelling waves at the origin. Bulk matter may well have a number of interconnected sources and sinks representing whole systems of particles and the analysis of rotating systems will be different in such cases, especially in the polar direction where the mutual interference may create rod-like overall symmetry, unlike the truncated but self-contained polar distributions described below. In searching for solutions of primary systems with origins at a single point, we are guided by the need to satisfy two basic criteria: (i) the effect of Coriolis acceleration upon the trajectories of all components of the wave system and (ii) the phase-locking requirement that the journey time of all component trajectories of the wave must be identical.

This stable phase-locking, apart from giving rise to rigidity and proper time-keeping, results in an important non-linearity of the space. A particle which is created following a cataclasmic event is built up from the interference of wave systems which have equal measures of wave distance along the circular paths and this creates an 'absolute' non-linearity in the radial measure. The radius of the system is contracted relative to that which would hold in the absence of rotation. For a non-rotating system the null lines would be spatially linear. They could only interfere with their own reflections from some physical boundary and the radial distances would then be linear. The helical null lines, on the other hand, build a system from mutual interference where the equal wave distances on the arcs result in a compression of the distance measured in the usual radial direction. This cramping of the radial measure is progressively greater towards the limb. It is partly on this account that some of the phenomena described in this paper may appear strange if one endeavours to interpret them in terms, for example, of linear electromagnetism or the traditional relativistic transformations which assume a linear system. This radial non-linearity will be discussed again in § 8. Let us suggest, for the present, that the whole system may be formed from the energy contained in linear fields which have suffered a cataclasmic event forcing them into a condensed rotating state from which, as we shall see, they cannot escape.

We have investigated the structure of three-dimensional clocks based upon these principles by constructing three-dimensional models which appear to satisfy all the requirements and we have then endeavoured to interpret these models in an elementary mathematical form. Although this method of deduction is not often encountered in modern physics, we believe that it is better, in this account, to follow the interpretation of these models rather than to labour through a more abstract derivation which might risk both error and confusion.

The models are dependent upon the flux distribution towards the poles but are all subject to the overriding requirement, originally derived from the treatment of phase-locked cavities but equally applicable to ideal clocks, that the radiation must be perfectly coherent under all circumstances at all times. All the constituents which contribute to the flow of information must operate coherently and return to the origin with a coherent and perfectly regular periodicity.

Three models which have this property will now be discussed; they have the outer forms of a sphere, an ellipsoid having a $\pi/2$ ratio of major (polar) to minor (equatorial) axes and a torus.

4. A spherical isotropic clock

In the isotropic model the complete projections of the helical null lines occur at all angles of θ but wrap around the polar axis in such a manner that all the path lengths remain identical and the signals remain in phase at equal distance from the origin. This model is governed by the equation in spherical polar coordinates centred on O

$$r = R_0 \sin[(\phi - \alpha) \sin \theta] \qquad \text{or} \qquad \phi = \alpha + \operatorname{cosec} \theta \sin^{-1}(r/R_0). \tag{4}$$

In practical terms, this path may be visualised by taking a piece of paper upon which are inscribed two circles, as shown in figure 5, and then cutting along the broken line tangent to the small circle at the origin O, and also around the circumference of the large circle. If one now overlaps the paper at the tangential incision one forms a cone upon which the smaller circle is inscribed with one point touching the origin and the

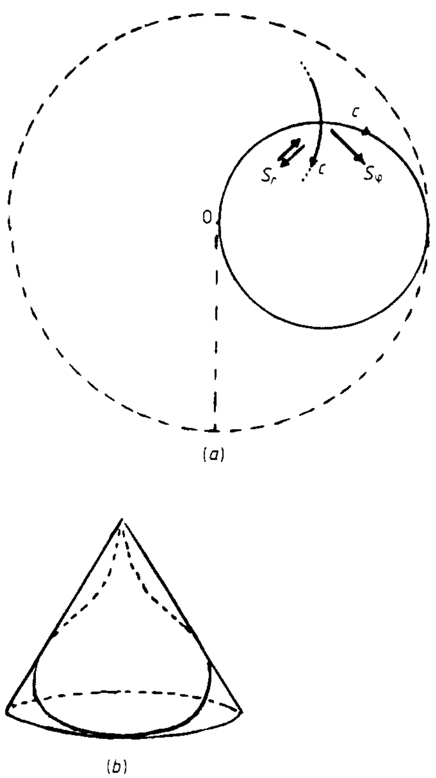

Figure 5. The projection of a helical null line in an isotropic clock. Mark a piece of paper with the circles and line as shown in (a). Cut along the broken lines and form into a cone of half angle θ. A typical projected helical null line then appears as shown in (b). The vector addition in 3-space is also clearly seen in this construction.

other diametrically opposite point defining an extremity where $r = R_0$. The half-angle of the cone is θ in equation (4).

We now define an elementary flux vector $\boldsymbol{S}$ in the direction of a single projected helical null line. This gives, for each null line, two radial components of flux which are equal in magnitude but opposite in direction, given by the roots of the expression

$$|\boldsymbol{S}|(1-r^2/R_0^2)^{1/2}.$$

For the tangential component, there are equal components of flux pointing in the same direction and each of magnitude

$$S_\phi = |\boldsymbol{S}|r/R_0.$$

We are interested in the total flux flowing on similar paths at all angles α about the axis. We may intercept the flux in an elementary circular hoop at r and θ with a cross sectional area of $\delta A = \delta r \times r\delta\theta$. The angular velocity of this circular hoop is dependent upon the value of θ

$$\Omega_\theta = \varsigma/(R_0 \sin\theta).$$

Thus, integrating the energy in the elemental hoops over the volume of a sphere, we obtain the total rotational energy of the system

$$E_\phi = \int_0^\pi \int_0^{R_0} |\boldsymbol{S}| \frac{4\pi}{\varsigma} r^2 \sin\theta \,\mathrm{d}r\,\mathrm{d}\theta = \frac{8\pi R_0^3}{3\varsigma}|\boldsymbol{S}|$$

or

$$|\boldsymbol{S}| = 3\varsigma E_\phi / 8\pi R_0^3. \tag{5}$$

Thus the contributions to the flow of energy are

$$S_\phi = \frac{3\varsigma E_\phi}{4\pi R^3}\frac{r}{R} \qquad S_\theta = 0 \qquad S_r = 0. \tag{6}$$

We next compute the potential energy of the radial components of $\boldsymbol{S}$.

Consider a spherical shell at radius r. The normal component of flux arriving at this shell from the outer region of the sphere is

$$S_{r_1} = |\boldsymbol{S}|(1-r^2/R^2)^{1/2} 4\pi r^2$$

and there is a similar component S_{r_2} from the inner regions.

To obtain the potential energy δE_r we take the sum of the two contributions $(|S_{r_1}| + |S_{r_2}|)$ over the area of the shell and multiply by the transit time over the thickness of the shell, $\mathrm{d}r$. Now the transit time $= \mathrm{d}r/$radial velocity $= \mathrm{d}r/\varsigma(1-r^2/R_0^2)^{1/2}$.

Hence

$$E_r = \int \delta E_r = 2\int_{r=0}^{R_0} |\boldsymbol{S}| \frac{(1-r^2/R^2)^{1/2}}{\varsigma(1-r^2/R^2)^{1/2}} 4\pi r^2 \,\mathrm{d}r = \frac{8\pi r^3}{3\varsigma}|\boldsymbol{S}| = E_\phi. \tag{7}$$

The complete system is therefore conservative and stable.

The angular momentum is given by

$$P = \int_0^\pi \int_0^{R_0} |\boldsymbol{S}| \frac{8\pi R_0}{\varsigma^2} r^2 \sin^2\theta \,\mathrm{d}r\,\mathrm{d}\theta = \frac{4\pi^2 R_0^4}{3\varsigma^2}|\boldsymbol{S}|.$$

Substituting $|\boldsymbol{S}|$ from equation (5)

$$P = \frac{E_\phi \pi R_0}{2\varsigma} = \frac{E_\phi \pi}{2\Omega_0} = \frac{\pi E_{\mathrm{T}}}{4\Omega_0}. \tag{8}$$

Though this entity is relativistically rigid its surface state is unlike that of a classical 'rigid body' in which the surface velocity is a maximum at the equator and a minimum close to the poles. The particle has every part of its surface moving at the speed of light and yet it is perfectly spherical with a relativistically invariant distance from its surface to its centre and a fully defined axis of rotation through that centre. The proper time-keeping is fully defined in terms of the angular frequency Ω_0 at the equator; the angular frequency at any other latitude, $\Omega_0 = \varsigma/R \sin\theta$, is therefore also fully defined but increases towards the poles. Within the outer boundary defined by $R = \varsigma/\Omega_0$ the clock consists of pure non-particulate matter.

For the purposes of external reference, Ω_0 may not be directly observable but equation (8) shows that Ω_0 is fully defined from the angular momentum and total internal energy of the closed system. The angular momentum therefore constitutes a real observable indicator of proper time for the entity.

5. An ellipsoidal proper clock

This clock retains the property of equal transit time for all the projected helical null lines by utilising a $\pi/2$ ratio for the polar and equatorial axes. This implies that the equatorial 'ray paths' are circular, as in all these analyses, but the paths straight up the polar axis are infinitely thin 'hairpins' having the same total length. The paths at arbitrary latitudes are simply determined by Coriolis acceleration which aberrates the angles but maintains the local speed c. The form of the analysis is otherwise similar to that for the spherical clock and we will not repeat it here but it should be noted that it may be modified by a reduction of the flux density towards the poles so that the moment of inertia is increased and the resultant object appears to be stable. The polar reduction of flux may be achieved by reducing the number of 'flux lines' as distinct from reducing the velocity of flow which will be illustrated in the next (toroidal) example. As in the case of the sphere, the whole surface of the simple ellipsoid has a phase velocity of c in the chosen frame but the whole system rotates as a coherent body.

6. A toroidal proper clock

We will analyse the case where the primary wave system is equatorial and the decreasing flux carries less momentum at high latitude. This influences the confinement of the wave and we find, from our three-dimensional models, that all the information concerned with a single circular component is contained within a sphere of the same diameter, R_0, centred at a point $R_0/2$ from the origin of the disturbance. This sphere has some interesting geometrical properties which we shall make use of in the following account.

Consider a set of spherical polar coordinates r, θ, ϕ centred on the origin O. The sphere to which we refer has its centre at $r = R_0/2$, $\theta = \pi/2$, $\phi = \pi/2$. If we take any cone at polar angle θ, centred on the origin, the intersection of the cone with the sphere always lies upon the surface of a cylinder, one side of which is contiguous with the polar axis.

Consider an element of the radiating disturbance which emanates at an angle θ to the polar axis. This moves out upon the surface of the cone along the line where it intersects the spherical surface and, as seen projected into the equatorial plane, it appears to form a circle, around the circumference of the imaginary cylinder, touching the origin but of smaller diameter ρ than R_0 (see figure 6). This circle is traced out in exactly the same time as that of the larger circle; indeed, the parameter $\Omega = \mathrm{d}\phi/\mathrm{d}t$ is identical for all such circles, in accordance with the required coherence of phase and this property holds throughout the system. We find that the geometry of the locus of any point on the wavefront can be described by the equation

$$r = R_0 \sin\theta \sin(\phi - \alpha) \tag{9}$$

where α can take all values between 0 and 2π. The system will then have the ultimate form of a torus of outer radius R_0 in which the inner radius is zero. This corresponds to the emission of a coherent wave symmetrically in all directions in the equatorial plane.

We shall now use this model as the basis of elementary analysis, hoping that its mysteries may become clear as the analysis progresses.

Let the radiation in the equatorial plane be characterised by an elementary flux vector $\boldsymbol{S}$. The direction of the vector is along the single circular path and its magnitude at the equator is $|\boldsymbol{S}|$. We are considering a model in which the flux decreases towards the poles and we find that for this model, in which the flux vector is directed along the paths defined by equation (9), the flux decreases towards the poles according to the relation

$$S(\theta) = |\boldsymbol{S}| \sin^2\theta. \tag{10}$$

This reduced flux which embarks on its journey on a particular conical surface remains upon that surface as it follows the path equation and the vector has no component

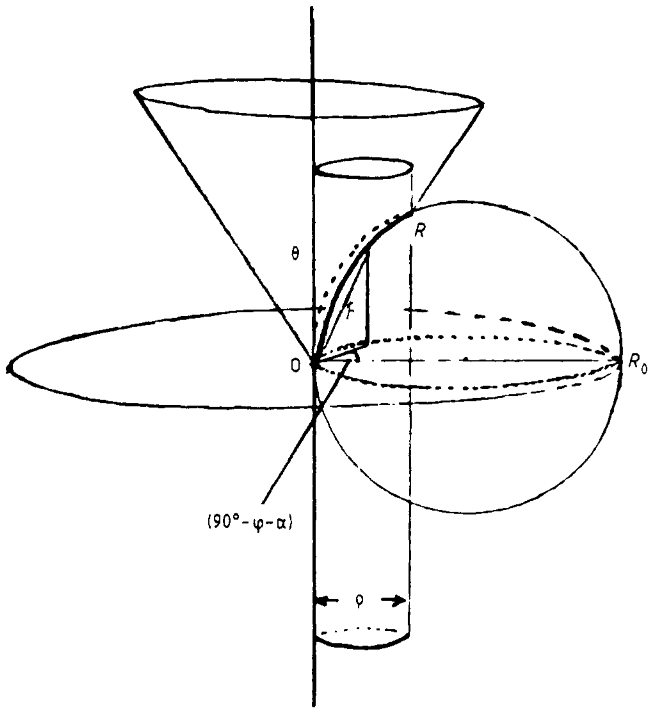

Figure 6. The locus of part of a wavefront in 3-space. The heavy line from O to R represents the locus of the wavefront through R. ϕ is difficult to portray in this view; its complement is shown.

out of this surface but only components in the dimensions of r and ϕ. These components will be identified by the subscripts S_r and S_ϕ. The geometry of the model shows that the path at any value of θ has a radial component on its outward journey and a symmetrical radial component on its inward journey and these are oppositely directed but each of magnitude

$$S_r = |\boldsymbol{S}| \sin^2 \theta[1-(r^2/R^2)]^{1/2} \tag{11}$$

where $R = R_0 \sin \theta$.

The model also shows that there are two components of tangential flux, $\boldsymbol{S}_\phi$, for each path and these point in the same direction, each having a magnitude of

$$S_\phi = |\boldsymbol{S}| \sin^2 \theta r/R. \tag{12}$$

Thus, for a single path emanating at a polar angle θ we have the following total contributions to the net flow of energy:

$$S_\theta = 0 \qquad S_\phi = 2|\boldsymbol{S}| \sin^2 \theta r/R \qquad S_r = 0. \tag{13}$$

There is, in addition, a contribution to the total energy from the radial components but this does not contribute a net flow and will later be seen to have an inert character.

We are now in a position to integrate the total flux flowing on similar paths at all angles, α, about the axis. In order to do this we take an elemental circular hoop at r and θ which has a cross section of $\delta A = \delta r \times r\delta\theta$. This hoop will capture energy from the flux, S_ϕ, of all the contributions throughout the range $\alpha = 0\text{–}2\pi$ and, as we have already seen, each element of $\boldsymbol{S}$ gives two contributions to S_ϕ in any one period, as in equation (13). The energy in a hoop is therefore

$$\delta E_\phi = S_\phi \delta A \, 2\pi/\Omega_0.$$

Integrating for all the hoops in the system and substituting $\Omega_0 = \varsigma/R_0$ we obtain

$$E_\phi = \int_0^\pi \int_0^{R=R_0 \sin\theta} 2|\boldsymbol{S}| \sin^2 \theta \frac{r^2}{R} \frac{2\pi}{\varsigma} R_0 \, \mathrm{d}r \, \mathrm{d}\theta = \frac{\pi^2 R_0^3}{2\varsigma} |\boldsymbol{S}|. \tag{14}$$

In a similar manner we may compute the energy of the inert radial component of energy formed from the interference of the oppositely directed radial components of flux

$$E_r = \int_0^\pi \int_0^{R=R_0 \sin\theta} 2|\boldsymbol{S}| \sin^2 \theta \frac{(1-r^2/R^2)^{1/2}}{U_r} 2\pi r \sin\theta \, \mathrm{d}r \, r \, \mathrm{d}\theta \tag{15}$$

where U_r is the velocity of the energy in the radial direction. But from the geometry of the model $U_r = \varsigma \sin^2 \theta (1-r^2/R^2)^{1/2}$. Therefore

$$E_r = \int_0^\pi \int_0^{R=R_0 \sin\theta} \frac{2}{\varsigma} |\boldsymbol{S}| \sin\theta \, 2\pi r^2 \, \mathrm{d}r \, \mathrm{d}\theta = \frac{\pi^2}{2\varsigma} R_0^3 |\boldsymbol{S}| = \frac{\pi^2 R_0^2}{2\Omega_0} |\boldsymbol{S}| \tag{16}$$

which is precisely equal to the energy in the tangential component and the system is therefore conservative.

The total energy is

$$E_\mathrm{T} = (\pi^2/\varsigma) R_0^3 |\boldsymbol{S}| = (\pi^2 R_0^2/\Omega_0) |\boldsymbol{S}| \tag{17}$$

and we may extract $|\boldsymbol{S}|$ in terms of E_T

$$|\boldsymbol{S}| = (\varsigma/\pi^2 R_0^3) E_\mathrm{T} = (\Omega_0/\pi^2 R_0^2) E_\mathrm{T}. \tag{18}$$

The angular momentum, P, of the complete system may be computed in a similar manner:

$$P = \int_0^\pi \int_0^{R=R_0 \sin\theta} \frac{8\pi}{\varsigma^2} |\boldsymbol{S}| \sin^2\theta \frac{r^2}{R} R_0^2 \,\mathrm{d}r\, \mathrm{d}\theta = \frac{\pi^2}{2\varsigma^2} R_0^4 |\boldsymbol{S}| = \frac{\pi^2 R_0^2}{2\Omega_0^2} |\boldsymbol{S}| \tag{19}$$

but, from equation (18),

$$|\boldsymbol{S}| = (\varsigma/\pi^2 R_0^3) E_{\mathrm{T}}$$

and therefore the angular momentum

$$P = (R_0/2\varsigma) E_{\mathrm{T}} = E_{\mathrm{T}}/2\Omega_0. \tag{20}$$

It will be appreciated that in all the preceding integrations we have implicitly integrated over all values of α and the offset sphere, associated with a helical null line at a single value of α, is now integrated around the axis to produce a resultant body in the form of a torus having outer radius R_0 and inner radius zero. This is shown in figure 7. The torus rotates in the manner of a classical rigid body.

Equation (20) may be rewritten

$$\Omega_0 = E_{\mathrm{T}}/2P. \tag{21}$$

But Ω_0 is a measure of proper time-keeping and this measure may be extracted from a knowledge of the total energy and angular momentum of the entity.

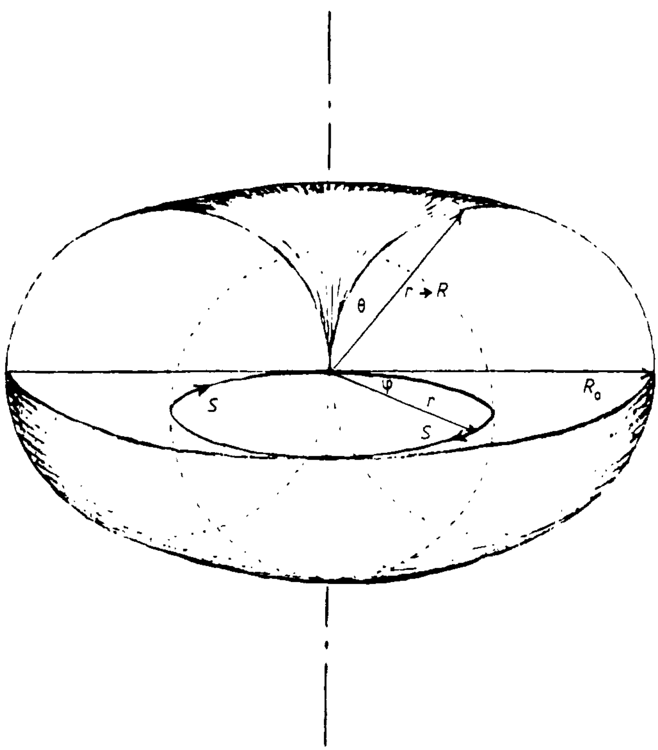

Figure 7. A segment removed to show three-quarters of a torus. This gives an elliptical view (full line) of a projected helical null line in the equatorial plane. The dotted circles show the limit of the structure out of the equatorial plane.

7. An interpretation of the analysis

The analysis shows that, if one of these configurations is formed, it may exist as a self-contained entity—it is a phase-locked structure which requires no containing walls. We may look upon the entity either as a wave system moving in and out from the centre on curved re-entrant trajectories or as a system with two components of energy, one of which, the radial component, is inert, whilst the other is a perfectly ordered rotating field which wheels around the central axis. Both of these interpretations refer to the 'cedilla' frame, i.e. the frame in which the helical null lines exist, but is this the inertial frame in which the centre is at rest? The physics of the whole situation may make more sense if the 'cedilla' frame is interpreted as a rotating rest frame, not unlike an inertial rest frame but applicable to a system which has an absolute rotation of Ω_0 at all parts. Thus

$$dx^2+dy^2+dz^2=c^2\,dt^2=ç^2\,dt^2 \tag{22}$$

where $|c|=|ç|$ but c refers to an inertial rest frame whereas $ç$ and its associate dimensions in x, y and z are measured in a rotating rest frame, i.e. a frame centred on O, with axes extending from O to R_0 and rotating with the same angular velocity as a rotating system in which the radial flux components are at rest. It therefore suffers absolute rotation with Ω_0 implanted throughout. The helical null lines then become null geodesics in rotating space. Thus, there is a physical difference between participation in a rotation and observation of a rotation from a coexistent inertial frame in which the coordinates of the centre of rotation are at rest. The concept of the 'cedilla' frame was inherent in the analysis used by Jennison (1963) to account for the Mössbauer results of Champeney and Moon and has subsequently appeared in many other analyses.

The trapped wave system may be identified as a solution for de Broglie (not Schrödinger) waves having phase velocity c or, possibly, electromagnetic waves with the same intrinsic velocity (Gueret and Vigier 1984, Jennison 1983). The de Broglie waves assume the quantum condition whilst the EM waves satisfy the condition without its assumption when the experimentally observed values are introduced (Jennison 1978, 1980, 1983). We therefore venture to interpret the entity in terms of a rotating de Broglie wavepacket which can equally be considered as an electromagnetic system, although we will defer a discussion of the latter point until later in this paper.

The trapped wave energy will, in these circumstances, exhibit an inertial rest mass, a result which may be derived from Einstein's second relativity paper (Einstein 1905) or, separately, from the theory of phase-locked cavities. A paper by Jennison (1980) contains many points relevant to the present discussion. The content of the region of space may therefore be ascribed the title 'matter' and be granted the relevant properties of matter even though the matter is pure and non-particulate in its consistency. Thus we find that we have produced a recipe for a body of pure matter which is relativistically rigid and self-supporting whilst it spins upon an axis in space.

The physical entity that we have derived appears to be a stable configuration of de Broglie waves of a particular wavelength. If the particle is essentially electromagnetic, such as an electron or proton, then it does not appear possible for the particle to contain two criminals each having the precise quantity of energy that is required and also having such similar properties and we consider that it is more likely that the de Broglie waves in these systems are essentially of the same substance as electromagnetic waves locked into a spinning configuration. If we wish to interpret the particle in terms of the photon concept, then the photon must have a precise wavelength corresponding to a multiple of the length of the projected helical null line in the equatorial plane, and it must also have a 'thickness' corresponding to the field distribution which gives rise to a depth out of the equatorial plane; it is therefore equivalent to the electromagnetic wave structure. If the entity were to be annihilated by a conjugate particle then one might expect the angular momenta to oppose but the total energy of each entity could be released as a free wave or photon at the wavelength of the trapped energy.

Let us assume that the annihilation energy is $h\nu$. This is not assuming the quantum theory but merely quoting the experimentally observed energy of annihilation of the electron and the proton. Equation (20) gives the angular momentum of the toroidal entity

$$P = E_T/2\Omega_0. \tag{23}$$

Substituting $E_T = h\nu$ we obtain

$$P = h\nu/2\Omega_0 \tag{24}$$

and putting $\Omega_0 = 2\pi\nu_0$

$$P = h\nu/4\pi\nu_0. \tag{25}$$

Thus if $\nu = \nu_0$ we have $P = \frac{1}{2}\hbar$, consistent with the angular momentum of the stable subatomic particles. This calculation may also be performed in terms of a half-wavelength, $\lambda_0/2$, trapped around each circular path in the equatorial plane, using the alternative form

$$P = (R_0/2c)E_T = (\lambda_0/4\pi c)E_T$$

and substituting $E_T = h\nu = hc/\lambda$,

$$P = \tfrac{1}{2}\hbar \tag{26}$$

if λ_0 is the free space annihilation wavelength, λ, i.e. the Compton wavelength. This, however, does not mean that the principles are restricted to subatomic particles but it indicates that, for its most basic clocks, nature selects entities corresponding to specific energies which are not derived in this paper except by implication from the de Broglie interpretation or from the experimental data.

8. Deductions and conclusions

We believe that we have shown that proper clocks may be formed, albeit under special conditions, from simple wave ingredients. We have not investigated the special conditions for formation but for the present we simply point to the remarkable processes of annihilation and pair production, to the formation of the ball lightning, and to certain cosmic phenomena. The clocks which we propose have the associated properties of rest mass, angular momentum, self-induced inertia and, in certain cases at least, monopolar electric charge. The entities have much in common with the phase-locked cavities which have been discussed in other papers and reference may be made to them for a fuller account of the inertial mechanism.

The proper time-keeping of the fundamental clocks, composed entirely from non-particulate matter, is given by the ratio of the total energy and the angular momentum of the entities.

We believe that we have also thrown a little light on the mysteries of absolute rotation. The energy of the rotation modifies the geometry of otherwise empty space for the space is no longer empty if the waves are trapped, for trapped waves, unlike freely propagating waves, have mass and inertia; indeed they are matter. The distortion of space in a rotating frame is non-linear and the projected non-linearity is not transformed away by the observation of that geometry in coexistent inertial space and to this we will add a further comment.

There are two incidental outputs of this analysis which are worthy of further remark. The first is the radial contraction of a relativistically rigid rotating system. It is not attributable to special relativity and we have been lucky in that our choice of a 'rigid' system has shed light on the phenomenon. A relativistically rigid system exhibits a real physical contraction of the mean radius. This contraction is given by $r = (c/\Omega)\sin(\Omega s/c)$, where s is the length of the wave mechanical 'light' path (circular arc) and corresponds to the length of the radius if the same system could exist in a non-rotating state. It appears in both the rotating frame (as the mean or 'integrated piecemeal' radius) and in the corresponding measurement of this parameter in the underlying inertial frame, since each element of this radius is normal to the local velocity vector and is not subject to Lorentz contraction. The radial contraction is an 'absolute' phenomenon which may be attributed very simply to the invariance of the length of the null lines which is imposed by the phase-locking requirement. This is equivalent to the construction of the three-dimensional spatial system from equal wave distances along the curved trajectories, resulting in a non-linear scale in the radial direction. The principle involved is also applicable to macroscopic non-rigid bodies but is masked by the very much larger effects of the elastic deformation, for which it may well be responsible. The point that we are making is that the metrical radius actually shrinks upon rotation in order to accommodate the curvature of the wave mechanical 'light' paths within the system. This phenomenon applies independently of the further radial (scale) contraction $r' = r(1-\Omega^2 r^2/c^2)^{1/2}$ which is measured entirely from a single rotating radius (e.g. by radar or parallax, Jennison (1964), Davies and Jennison (1975), Ashworth and Jennison (1976) and Jennison and Ashworth (1976)).

The second incidental outcome concerns the nature of the travelling wave components in the system. It has already been noted (Mackinnon 1981a, b, Guerét and Vigier 1984) that the electromagnetic 'phase-locked cavities' previously discussed by one of the authors (Jennison 1978, 1980, Jennison and Drinkwater 1977) satisfy the requirements for de Broglie 'solitons' if the wave mechanical disturbance is electromagnetic. We also note that trapped electromagnetic waves exhibit a rest mass and inertia in their own right (Jennison 1978, 1980, 1985). Furthermore, the remarkable processes of pair production and annihilation, at low collisional velocities, show that discrete particles having mass may transform to pure electromagnetic disturbances. We therefore now assert that, in our opinion, the wave mechanical disturbances which have been discussed herein are simple electromagnetic field systems in the unfamiliar form of travelling waves trapped in a spinning configuration which nature appears to permit at certain wavelengths (or equivalent energies) following a cataclasmic event. Indeed, if we follow through this argument and relate it to the experiment of one of us (Jennison 1982) we find that the motion of the travelling wave components relative to the metric of the rotating system is such that the electromagnetic field system may appear stationary in the inertial frame of the centre and thereby give rise to a diverging electrostatic field and a magnetic dipole, mocking the appearance of an electrically charged particle with magnetic moment, angular momentum and rest mass. The static nature of the external field which is produced from this configuration, notwithstanding the mechanical angular momentum, is such that there should be no tendency for the field energy to escape by radiation.

The divergence of the field vector may be seen very simply by sketching short orthogonal vectors at intersecting points on the 'ray paths' on figure 4 and noting that the mutual interference between the vectors on each outgoing and ingoing ray at the intersection results in only a radial component. There is also a further orthogonal pair available through the paper which may correspond to a magnetic component. The radial distributions of field follow figure 3 with a 'soft' outer boundary for odd half-integral values of n, corresponding to an accessible external field. The azimuthal distributions are symmetrical if all components are launched simultaneously in phase, as in § 3, but it is perhaps more likely that the phase-locking is conjugate diametrically across the axis. This gives a structure like the segments of an orange, dictated by the group wavelength $\pi r/n$.

We do not suggest that the system of helices is unique in the formation of a proper clock. For example, we have noted that a system of spirals which project into the equatorial plane with the form $r = R_0 \sin(\phi/2+\alpha)$ may also satisfy the requirements. These spirals enter and depart from the time axis along the same infinitesimal radius at the centre, as shown in figure 8.

We have performed a series of further experiments to investigate the field phenomena but these do not directly affect the efficacy of the system as proper clocks and they will be reported in another communication. The general principles may apply on the scale of the majority of fundamental particles, ball lightning, condensed cosmic matter, or even to a sphere of the Hubble radius. Indeed, one may venture to ask why empty 'inertial' space has a refractive index and can this possibly be related to the fact that the universe may be slowly rotating? From the wave mechanical treatment used in Jennison (1983) it may be argued that a slow rotation may account for the universality of $\hbar$. Nevertheless, nature may select only certain time pieces by the

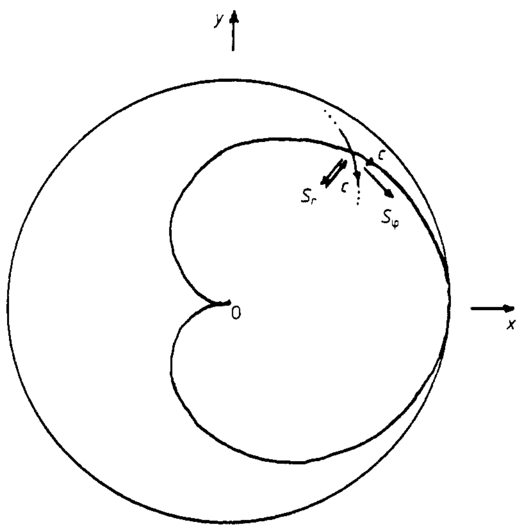

Figure 8. A second family of proper clocks. A curve having the equation $r = R\sin(\phi/2+\alpha)$ in xy (equatorial) space is the projection of a phase-locked system of spirals of pitch angle $1/c$ in 4-space which also appear to satisfy Minkowski's equation and produce a second family of proper clocks.

application of rules which have not been incorporated in this paper or have yet to be discovered.

Finally, we would say that we believe that proper clocks having the properties of matter may be formed from the interference of rotating de Broglie waves and if one dares to identify these waves with the same substance as electromagnetic waves, then the major properties of fundamental particles ensue.

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